Polynomial Equations

1. Linear Equation: $AX+B=0$ (1)

* If $A\neq 0$ then $X=-B/A$

*If $A=0$ then

-If $B\neq 0$ (1) has no solution.

-If $B=0$ (1) has infinite solutions.

2. Quadratic Equation: $AX^2+BX+C=0$ (2)

*If $A=0$ then (2)$\Leftrightarrow$(1)

*If $A\neq 0$ then

$(2) \Leftrightarrow X^2+(B/A)X+C/A=0$

$\Leftrightarrow (X+B/(2A))^2=(B^2-4AC)/(4A^2)=\Delta/(4A^2)$

-If $\Delta>0$ then $X=(-B\pm\sqrt{\Delta})/(2A)$

-If $\Delta=0$ then $X=-B/(2A)$

-If $\Delta<0$ then $X=(-B\pm i\sqrt{(-\Delta)})/(2A)$

3. Cubic Equation: $AX^3+BX^2+CX+D=0$ (3)

*If $A=0$ then (3)$\Leftrightarrow$(2)

*If $A\neq 0$ then let $X=Y-B/(3A)$

Then (3) becomes $Y^3+pY+q=0$ (*)

where p and q are constants.

Let $Y=u+v$ then (*) becomes $(u+v)^3+p(u+v)+q=0$

Or $u^3+v^3+q+(u+v)(3uv+p)=0$

Take $u^3+v^3+q=0$ and $3uv+p=0$

Then we get u^3 and v^3 are roots of equation: $t^2+qt-p^3/27=0$

After we determine the solution(s) for t, suppose we can set the solution(s) as:

$u=r.e^{i(\theta/3+2\pi k/3)}$ and $v=r.e^{i(-\theta/3+2\pi k`/3)}$

Or $u=u_o.e^{i.2\pi k/3}$ and $v=v_o.e^{i.2\pi k`/3}$

Then $Y=u+v=u_o.e^{i.2\pi k/3}+v_o.e^{i.2\pi k`/3}$

*If $k\equiv k`$ (mod 3) then $Y=(u_o+v_o)e^{i.2\pi k/3}$

It implies that Y^3 is a real number; hence, Y must be required to be REAL.

It forces $k\equiv k`\equiv 0$ (mod 3) and $Y_1=u_o+v_o$

*If $k\equiv k`\pm 1$ (mod 3) then $Y_2=j.u_o+j^2.v_o$, $Y_3=j^2.u_o+j.v_o$

where $j^3=1$ or $j=-½+i\sqrt{3}/2$

4. Quartic Equation: $AX^4+BX^3+CX^2+DX+E=0$ (4)

*If $A=0$ then (4)$\Leftrightarrow$(3)

*If $A\neq 0$ then let $X=Y-B/(4A)$

Then (4) becomes $Y^4+pY^2+qY+r=0$ or $Y^4+pY^2=-qY-r$

But $(Y^2+p)^2=Y^4+2pY^2+p^2=(-qY-r)+pY^2+p^2$

Or $(Y^2+p)^2+2z(Y^2+p)+z^2=(-qY-r)+pY^2+p^2+2z(Y^2+p)+z^2$

Where z is unknown

Or $(Y^2+p+z)^2=(p+2z)Y^2-qY+(p^2+z^2+2pz-r)$ (**)

We will make the right hand side be a perfect square.

Then put $\Delta=q^2-4(p+2z)(p^2+z^2+2pz-r)=0$

Or $8z^3+20pz^2+(16p^2-8r)z+(4p^3-4pr-q^2)=0$

Solve this equation for z; hence (**) becomes $(Y^2+p+z)^2=(mY+n)^2$

<u>*Second method*</u>

Let $(z^2+kz+m)(z^2-kz+n)\equiv z^4+pz^2+qz+r$

Or $z^4+(m+n-k^2)z^2+k(n-m)z+mn\equiv z^4+pz^2+qz+r$

Then we put $m+n=p+k^2$, $n-m=q/k$, $m.n=r$ *where m, n and k are unknown*

Then $m=(p+k^2-q/k)/2$ *and* $n=(p+k^2+q/k)/2$ *but* $m.n=r$

Then we get $k^6+2pk^4+(p^2-4r)k^2-q^2=0$ *(cubic equation of* k^2*)*

Any root $k^2\neq 0$ *produces a pair of quadratic equations:*

$z^2+kz+(p+k^2-q/k)/2=0$ *and* $z^2-kz+(p+k^2+q/k)/2=0$

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