

Polynomial Equations

1. Linear Equation: $AX+B=0$ (1)

*If $A \neq 0$ then $X = -B/A$

*If $A = 0$ then

-If $B \neq 0$ (1) has no solution.

-If $B = 0$ (1) has infinite solutions.

2. Quadratic Equation: $AX^2+BX+C=0$ (2)

*If $A = 0$ then (2) \Leftrightarrow (1)

*If $A \neq 0$ then

(2) $\Leftrightarrow X^2 + (B/A)X + C/A = 0$

$\Leftrightarrow (X + B/(2A))^2 = (B^2 - 4AC)/(4A^2) = \Delta/(4A^2)$

-If $\Delta > 0$ then $X = (-B \pm \sqrt{\Delta})/(2A)$

-If $\Delta = 0$ then $X = -B/(2A)$

-If $\Delta < 0$ then $X = (-B \pm i\sqrt{-\Delta})/(2A)$

3. Cubic Equation: $AX^3+BX^2+CX+D=0$ (3)

*If $A = 0$ then (3) \Leftrightarrow (2)

*If $A \neq 0$ then let $X = Y - B/(3A)$

Then (3) becomes $Y^3 + pY + q = 0$ (*)

where p and q are constants.

Let $Y = u + v$ then (*) becomes $(u+v)^3 + p(u+v) + q = 0$

Or $u^3 + v^3 + q + (u+v)(3uv+p) = 0$

Take $u^3 + v^3 + q = 0$ and $3uv + p = 0$

Then we get u^3 and v^3 are roots of equation: $t^2 + qt - p^3/27 = 0$

After we determine the solution(s) for t , suppose we can set the solution(s) as:

$u = r \cdot e^{i(\theta/3 + 2\pi k/3)}$ and $v = r \cdot e^{i(-\theta/3 + 2\pi k'/3)}$

Or $u = u_0 \cdot e^{i \cdot 2\pi k/3}$ and $v = v_0 \cdot e^{i \cdot 2\pi k'/3}$

Then $Y = u + v = u_0 \cdot e^{i \cdot 2\pi k/3} + v_0 \cdot e^{i \cdot 2\pi k'/3}$

*If $k \equiv k' \pmod{3}$ then $Y = (u_0 + v_0) e^{i \cdot 2\pi k/3}$

It implies that Y^3 is a real number; hence, Y must be required to be REAL.

It forces $k \equiv k' \equiv 0 \pmod{3}$ and $Y_1 = u_0 + v_0$

*If $k \equiv k' \pm 1 \pmod{3}$ then $Y_2 = j \cdot u_0 + j^2 \cdot v_0$, $Y_3 = j^2 \cdot u_0 + j \cdot v_0$

where $j^3 = 1$ or $j = -1/2 + i\sqrt{3}/2$

4. Quartic Equation: $AX^4+BX^3+CX^2+DX+E=0$ (4)

*If $A = 0$ then (4) \Leftrightarrow (3)

*If $A \neq 0$ then let $X = Y - B/(4A)$

Then (4) becomes $Y^4 + pY^2 + qY + r = 0$ or $Y^4 + pY^2 = -qY - r$

But $(Y^2 + p)^2 = Y^4 + 2pY^2 + p^2 = (-qY - r) + pY^2 + p^2$

Or $(Y^2 + p)^2 + 2z(Y^2 + p) + z^2 = (-qY - r) + pY^2 + p^2 + 2z(Y^2 + p) + z^2$

Where z is unknown

$$\text{Or } (Y^2+p+z)^2 = (p+2z)Y^2 - qY + (p^2+z^2+2pz-r) \quad (**)$$

We will make the right hand side be a perfect square.

$$\text{Then put } \Delta = q^2 - 4(p+2z)(p^2+z^2+2pz-r) = 0$$

$$\text{Or } 8z^3 + 20pz^2 + (16p^2 - 8r)z + (4p^3 - 4pr - q^2) = 0$$

$$\text{Solve this equation for } z; \text{ hence } (**) \text{ becomes } (Y^2+p+z)^2 = (mY+n)^2$$

Second method

$$\text{Let } (z^2+kz+m)(z^2-kz+n) \equiv z^4 + pz^2 + qz + r$$

$$\text{Or } z^4 + (m+n-k^2)z^2 + k(n-m)z + mn \equiv z^4 + pz^2 + qz + r$$

Then we put $m+n=p+k^2$, $n-m=q/k$, $m.n=r$ where m , n and k are unknown

$$\text{Then } m = (p+k^2 - q/k)/2 \quad \text{and} \quad n = (p+k^2 + q/k)/2 \quad \text{but } m.n=r$$

$$\text{Then we get } k^6 + 2pk^4 + (p^2 - 4r)k^2 - q^2 = 0 \quad (\text{cubic equation of } k^2)$$

Any root $k^2 \neq 0$ produces a pair of quadratic equations:

$$z^2 + kz + (p+k^2 - q/k)/2 = 0 \quad \text{and} \quad z^2 - kz + (p+k^2 + q/k)/2 = 0$$

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